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Cycle Related Power Cordial Graph

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Abstract – Let G= (V,E) be a graph with p vertices and q edges. Let X={-1 , 0 ,1 }, a Power Cordial Labeling of a Graph G with vertex set V is a bijection from V to a^x, x ∈ X and a ≥ 0 such that each edge uv is assigned with the label f(uv) = f(u).f(v) with the condition that the number of edges labeled with a^{-1}, the number of edges labeled with a^0 and the number of edges labeled with a^1 differ by atmost 1. The graph that admits a Power Cordial Labeling is called Power Cordial Graph. In this paper, we proved that cycle related graphs [P_n : C_3]and Double Triangular Snake $C_2(P_n)$are Power Cordial Graphs.

Keywords –PowerCordial Graph, Power Cordial Labeling.

2000 Mathematics Subject classification 05C78.

I. Introduction

A Graph G is a finite nonempty set of object called vertices together with a set of unordered pairs of distinct vertices of G which is called edges.Each pair e = {u,v} of vertices in E is called edges or a line of G.In this paper, we proved that cycle related graphs are Power Cordial Graphs. For graph theory terminology, we follow [2].

II. Preliminaries

Let G= (V,E) be a graph with p vertices and q edges. Let X={-1 , 0 , 1 }, a Power Cordial Labeling of a Graph G with vertex set V is a bijection from V to a^x, x ∈ X and a ≥ 0 such that each edge uv is assigned with the label f(uv) = f(u).f(v) with the condition that the number of edges labeled with a^{-1}, the number of edges labeled with a^0 and the number of edges labeled with a^1 differ by atmost 1.

The graph that admits a Power Cordial Labeling is called Power Cordial Graph. In this paper, we proved that cycle related graphs [P_n : C_3] and Double Triangular Snake $C_2(P_n)$are Power Cordial Graphs.

Definition:2.1

[P_n : C_3] is a graph obtained from a path P_n by joining one vertex of a cycle $C_3$to every vertex of a path.

Definition:2.2

A Double Triangular Snake $C_2(P_n)$ is obtained from the path $(v_1, v_2, v_3, \ldots\ldots v_n)$ by replacing every edge by two triangles C_3.

III. Main Results

Theorem: 3.1

$[P_n : C_3](n \geq 2)$is a Power Cordial Graph.

Proof:

Let the graph G be $[P_n : C_3]$.

Let $V(G)=\{[u_i : 1 \leq i \leq n], [v_{i1}v_{i2} : 1 \leq i \leq n]\}$

$E(G)=\{[(u_iu_{i+1}) : 1 \leq i \leq n-1] \cup [(u_iv_{ij}) : 1 \leq i \leq n, 1 \leq j \leq 2] \cup [(v_{i1}v_{i2}) : 1 \leq i \leq n]\}$

Define $f : V(G) \to \{a^{-1}, a^0, a^1\}$

The vertex labeling are

$$f(u_i) = \begin{cases} a^0 & i \equiv 1(mod3) \\ a^1 & i \equiv 2(mod3) \\ a^{-1} & i \equiv 0(mod3) \end{cases} \quad 1 \leq i \leq n$$

$$f(v_{i1}) = \begin{cases} a^0 & i \equiv 0(mod3) \\ a^1 & i \equiv 1(mod3) \\ a^{-1} & i \equiv 2(mod3) \end{cases} \quad 1 \leq i \leq n$$

$$f(v_{i2}) = \begin{cases} a^0 & i \equiv 2(mod3) \\ a^1 & i \equiv 0(mod3) \\ a^{-1} & i \equiv 1(mod3) \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are

$$f^*(u_iu_{i+1}) = \begin{cases} a^0 & i \equiv 2(mod3) \\ a^1 & i \equiv 1(mod3) \\ a^{-1} & i \equiv 0(mod3) \end{cases} \quad 1 \leq i \leq n-1$$

$$f^*(u_iv_{i1}) = \begin{cases} a^0 & i \equiv 0(mod3) \\ a^1 & i \equiv 1(mod3) \\ a^{-1} & i \equiv 2(mod3) \end{cases} \quad 1 \leq i \leq n$$

$$f^*(u_iv_{i2}) = \begin{cases} a^0 & i \equiv 2(mod3) \\ a^1 & i \equiv 0(mod3) \\ a^{-1} & i \equiv 1(mod3) \end{cases} \quad 1 \leq i \leq n$$

$$f^*(v_iv_{i+1}) = \begin{cases} a^0 & i \equiv 1(mod3) \\ a^1 & i \equiv 0(mod3) \\ a^{-1} & i \equiv 2(mod3) \end{cases} \quad 1 \leq i \leq n$$

And, When n = 3m+1, m>0

$e_f(a^0)=e_f(a^{-1}) = e_f(a^1) = 4m+1, m>0$

When n = 3m-1, m>0

$e_f(a^0)=e_f(a^{-1}) = 4m-2$ and $e_f(a^1) = 4m-1, \ m>0$

When n = 3m, m>0

$e_f(a^0)=e_f(a^1) = 4m$ and $e_f(a^{-1}) = 4m-1, \ m>0$

Therefore the graph G satisfies the condition $|e_f(a^i) - e_f(a^j)| \leq 1$

Hence $[P_n:C_3]$ is a Power Cordial Graph.

For example, the Power Cordial labeling of $[P_3:C_3]$ is shown in figure 3.2

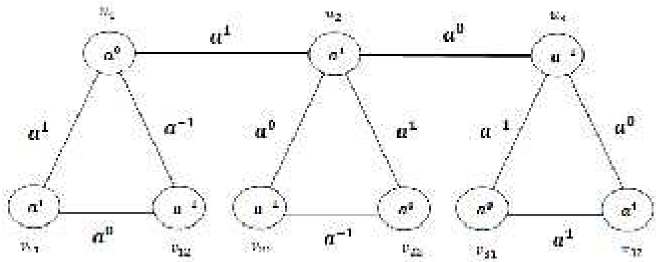

Figure 3.2

Theorem: 3.3

Double Triangular Snake $C_2(P_n)$ is a Power Cordial Graph.

Proof:

Let $V(C_2(P_n)) = \{[u_i : 1 \leq i \leq n], [v_i w_i : 1 \leq i \leq n-1]\}$

$$E(C_2(P_n)) = \{[(u_i v_i) \cup (u_i w_i) \cup (u_{i+1} v_i) \cup (u_{i+1} w_i) : 1 \leq i \leq n] \cup [(u_i u_{i+1}) : 1 \leq i \leq n-1]\}$$

Define f : V(G) → $\{a^{-1}, a^0, a^1\}$

The vertex labeling are

$$f(u_i) = \begin{cases} a^0 & i \equiv 0(mod3) \\ a^1 & i \equiv 2(mod3) \\ a^{-1} & i \equiv 1(mod3) \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} a^0 & i \equiv 1(mod3) \\ a^1 & i \equiv 0(mod3) \\ a^{-1} & i \equiv 2(mod3) \end{cases} \quad 1 \leq i \leq n-1$$

$$f(v_i) = a^0 \quad 1 \leq i \leq n-1$$

The induced edge labeling are

$$f^*(u_i u_{i+1}) = \begin{cases} a^0 & i \equiv 1(mod3) \\ a^1 & i \equiv 2(mod3) \\ a^{-1} & i \equiv 0(mod3) \end{cases} \quad 1 \leq i \leq n-1$$

$$f^*(u_i v_i) = \begin{cases} a^0 & i \equiv 2(mod3) \\ a^1 & i \equiv 0(mod3)\ 1 \le i \le n \\ a^{-1} & i \equiv 1(mod3) \end{cases}$$

$$f^*(u_i w_i) = \begin{cases} a^0 & i \equiv 0(mod3) \\ a^1 & i \equiv 2(mod3)\ 1 \le i \le n \\ a^{-1} & i \equiv 1(mod3) \end{cases}$$

$$f^*(u_{i+1} v_i) = \begin{cases} a^0 & i \equiv 0(mod3) \\ a^1 & i \equiv 1(mod3)\ 1 \le i \le n \\ a^{-1} & i \equiv 2(mod3) \end{cases}$$

$$f^*(u_{i+1} w_i) = \begin{cases} a^0 & i \equiv 2(mod3) \\ a^1 & i \equiv 1(mod3)\ 1 \le i \le n \\ a^{-1} & i \equiv 0(mod3) \end{cases}$$

And, When n = 3m+1, m>0

$e_f(a^0)=e_f(a^{-1}) = e_f(a^1) = 5m, m > 0$

When n = 3m, m>0

$e_f(a^0)=e_f(a^{-1}) = 5m-2$ and $e_f(a^1) = 5m-1,\ m > 0$

When n = 3m+2, m>0

$e_f(a^1)=e_f(a^{-1}) = 5m+2$ and $e_f(a^0) = 5m+1,\ m > 0$

Therefore the graph $C_2(P_n)$ satisfies the condition $|e_f(a^i) - e_f(a^j)| \le 1$

Hence $C_2(P_n)$ is a Power Cordial Graph.

For example, the Power Cordial labeling of $C_2(P_5)$ is shown in figure 3.4

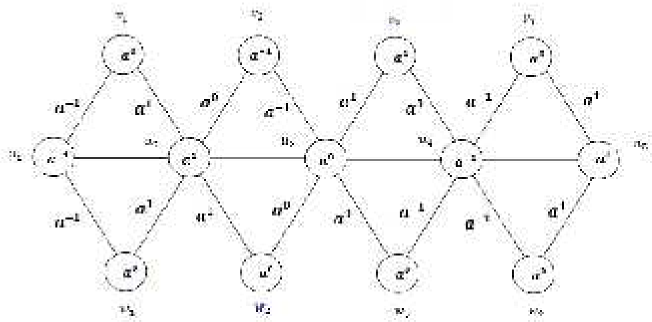

Figure 3.4

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