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Additive Square Mean Lebeling of Path Related Graphs

A.Nellai Murugan Department of Mathematics, V.O.Chidambaram College, Thoothukudi, anellai.vocc@gmail.com

P.Iyadurai Selvaraj Department of Computer Science, V.O.Chidambaram College, Thoothukudi, iyaduraiselvaraj@gmail.com

Abstract - Let G = (V,E) be a graph with p vertices and q edges. An additive square mean labeling of a Graph G with vertex set V = {0,1,2…n} such that if each edge uv is assigned the label $f(uv) = \frac{f(u)^2 + f(v)^2}{2}$ if $f(u)^2 + f(v)^2$ is even and $f(uv) = \frac{f(u)^2 + f(v)^2 + 1}{2}$ if $f(u)^2 + f(v)^2$ is odd and the edge labels are distinct.

The graph that admits a additive square mean labeling is called a additive square mean graph (ASMG). In this paper, we proved that Path P_n, Comb (Pn ⊙ K1), Double sided Comb($P_n \Theta 2K_1$) and $P_n \Theta 2P_m$ are additive square mean graphs.

Key words : *Additive square mean cordial labeling, Additive square mean cordial graph.*

2000 Mathematics Subject Classification 05C78.

1. Introduction :

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {uv} of vertices in E is called an edge or a line of G. In this paper , we proved that Path P_n, Comb (Pn ⊙ K1), Double sided Comb ($P_n \Theta 2K_1$) and $P_n \Theta 2P_m$ are additive square mean graphs.

2. Preliminaries :

Let G = (V,E) be a graph with p vertices and q edges. An additive square mean labeling of a Graph G with vertex set V = {0,1,2…n} such that if each edge uv is assigned the label $f(uv) = \frac{f(u)^2 + f(v)^2}{2}$ if $f(u)^2 + f(v)^2$ is even and $f(uv) = \frac{f(u)^2 + f(v)^2 + 1}{2}$ if $f(u)^2 + f(v)^2$ is odd and the edge labels are distinct.

The graph that admits a additive square mean labeling is called a additive square mean graph (ASMCG). In this paper, we proved Path P_n, Comb (Pn ⊙ K1), Double sided Comb ($P_n \Theta 2K_1$) and $P_n \Theta 2P_m$ are additive square mean graphs.

Definition 2.1 – Path

A graph with sequence of vertices u_1, u_2 …. u_n such that successive vertices are joined with an edge, P_n is a path of length n-1.

Definition 2.2 – Comb (Pn ⊙ K1)

It is a graph obtained from a path Pn by joining a pendent vertex to each vertices of Pn, it is denoted by Pn ⊙ K1

Definition 2.3 – Double sided Comb ($P_n \Theta 2K_1$)

It is a graph obtained from a path Pn by joining two pendent vertex to each vertices of Pn, it is denoted by Pn ⊙ 2K1

Definition 2.4 – $P_n \Theta 2P_m$

It is a graph obtained from a path Pn by joining two paths of length m, that is P_m and it is denoted by $P_n \Theta 2P_m$

3. Main results

Theorem 3.1

Path P_n is an additive square mean graph.

Proof

Let G be P_n

Let $V(G) = \{u_i : 1 \leq i \leq n\}$ and

E$(G) = \{(u_i\, u_{i+1}) : 1 \leq i \leq n-1\}$

Define $f : V(G) \rightarrow \{0,1,2 \ldots n\}$

The vertex labeling are

$f(u_i) = \{i, 0 \leq i \leq n-1\}$

The induced edge labeling are

$f^*(u_i u_{i+1}) = \{i * (i-1) + 1, \quad 1 \leq i \leq n-1\}$

The induced edge labels are ranging from 1 to (n-1)*(n-2)+1

Hence, P_n is an additive square mean graph

For example, P_5 is an additive square mean graph as shown in figure 3.1.

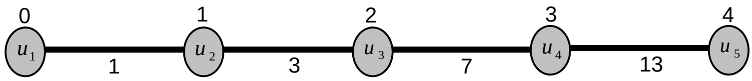

Figure 3.1

Theorem 3.2

Comb (Pn ⊙ K1) is an additive square mean graph.

Proof

Let G be Pn ⊙ K1

Let $V(G) = \begin{Bmatrix} u_i : 1 \le i \le n \\ v_i : 1 \le i \le n \end{Bmatrix}$ and

$E(G) = \{[(u_i\, u_{i+1}) : 1 \le i \le n-1] \cup [(u_i\, v_i) : 1 \le i \le n]\}$

Define $f : V(G) \to \{0,1,2 \ldots 2n\}$

The vertex labeling are

$f(u_i) = \{2(i-1)+1, 1 \le i \le n\}$ and

$f(v_i) = \{2(i-1),\ 1 \le i \le n\}$

The induced edge labeling are

$f^*(u_i u_{i+1}) = \{(2i+1) * (2i-1) + 2, \quad 1 \le i \le n-1\}$

and $f^*(v_i u_i) = \{(2i-1) * (2i-2) + 1, \quad 1 \le i \le n\}$

The induced edge labels are ranging from 1 to (2n-1)*(2n-2)+1

Hence, Pn ⊙ K1 is an additive square mean graph

For example, P5 ⊙ K1 is an additive square mean graph as shown in figure 3.2.

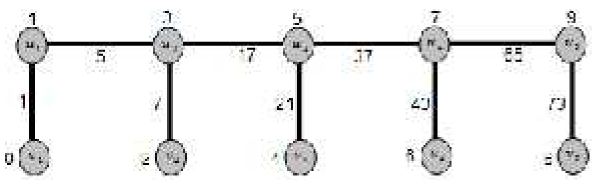

Figure 3.2

Theorem 3.3

Double sided Comb (Pn ⊙ 2K1) is an additive square mean graph.

Proof

Let G be Pn ⊙ 2K1

Let $V(G) = \begin{Bmatrix} u_i : 1 \le i \le n \\ v_{1i} : 1 \le i \le n \\ v_{2i} : 1 \le i \le n \end{Bmatrix}$ and

E$(G) = \{[(v_i\, v_{i+1}): 1 \le i \le n-1] \cup [(v_i\, v_{1i}): 1 \le i \le n] \cup [(v_i\, v_{2i}): 1 \le i \le n]\}$

Define $f: V(G) \to \{0,1,2 \ldots 3n\}$

The vertex labeling are

$f(v_i) = \{3(i-1)+1, 1 \le i \le n\}$,

$f(v_{1i}) = \{3(i-1),\ 1 \le i \le n\}$ and

$f(v_{2i}) = \{3(i-1)+2,\ 1 < i \le n\}$

The induced edge labeling are

$f^*(v_i v_{i+1}) = \{9((i-1)^2+1) + 15(i-1), \quad 1 \le i \le n-1\}$

$f^*(v_1 v_{1i}) = \{(i-1) * (9(i-1)+3)+1, \quad for\ 1 \le i \le n\}$

$f^*(v_1 v_{2i}) = \{9i(i-1)+3, \quad for\ 1 \le i \le n\}$

The induced edge labels are ranging from 1 to (3n-1)*(3n-2)+1

Hence, Pn ⊙ 2K1 is an additive square mean graph

For example, P_4 ⊙ 2K1 is an additive square mean graph as shown in figure 3.3.

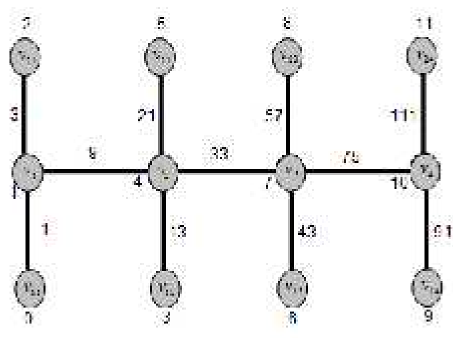

Figure 3.3

Theorem 3.4

$P_n \Theta 2P_m$ is an additive square mean graph.

Proof

Let G be $P_n \Theta 2P_m$

Let x be $(m-1)$

Let $V(G) = \begin{Bmatrix} u_i : 1 \le i \le n, \\ v_{ij} : 1 \le i \le n, 1 \le j \le x, \\ w_{ij} : 1 \le i \le n, 1 \le j \le x \end{Bmatrix}$ and

$E(G) = \{[(u_i u_{i+1}) : 1 \le i \le n-1] \cup [(v_{ij} v_{ij+1}) : 1 \le i \le n, 1 \le j \le x-1] \cup [(w_{ij} w_{ij+1}) : 1 \le i \le n, 1 \le j \le x-1] \cup [(u_i v_{i1}) : 1 \le i \le n] \cup [(u_i w_{ix}) : 1 \le i \le n]\}$

Define $f : V(G) \to \{0,1,2 \ldots (2x * n) + n\}$

The vertex labeling are

$$f(u_j) = \{(x * n) + (i-1),\ 1 \le i \le n\}$$

$$f(w_{ij}) = \{(j-1) * n + (i-1),\ 1 \le i \le n, 1 \le j \le x\}$$

$$f(v_{ij}) = \{[(x * n) + n] + [(j-1) * n] + (i-1),\ 1 \le i \le n, 1 \le j \le x\}$$

The induced edge labeling are

$$f^*(u_i u_{i+1}) = \{[(x * n) + (i-1)] * [(x * n) + i] + 1,\ 1 \le i \le n-1\}$$

If n is odd,

$$f^*(w_{ij} w_{i(j+1)}) = \left\{[(j-1) * n + (i-1)] * [(n * j) + (i-1)] + \frac{n^2+1}{2}, 1 \le i \le n, i \le j \le x-1\right\}$$

$$f^*(w_{ix} u_{i}) = \left\{[(x-1) * n + (i-1)] * [(n * x) + (i-1)] + \frac{n^2+1}{2}, 1 \le i \le n\right\}$$

$$f^*(v_{ij} v_{i(j+1)}) = \left\{[(x * n + n) + (j-1) * n + (i-1) * [(x * n + n) + (n * j) + (i-1)] + \frac{n^2+1}{2}, 1 \le i \le n, i \le j \le x-1\right\}$$

$$f^*(u_i v_{i1}) = \left\{[x * n + (i-1)] * [(n * x + n) + (i-1)] + \frac{n^2+1}{2}, 1 \le i \le n\right\}$$

If n is even,

$$f^*(w_{ij}w_{i(j+1)}) = \left\{[(j-1)*n+(i-1)]*[(n*j)+(i-1)]+\frac{n^2}{2}, 1 \le i \le n, i \le j \le x-1\right\}$$

$$f^*(w_{ix}u_i) = \left\{[(x-1)*n+(i-1)]*[(n*x)+(i-1)]+\frac{n^2}{2}, 1 \le i \le n\right\}$$

$$f^*(v_{ij}v_{i(j+1)}) = \left\{[(x*n+n)+(j-1)*n+(i-1)] * [(x*n+n)+(n*j)+(i-1)]+\frac{n^2}{2}, 1 \le i \le n, i \le j \le x-1\right\}$$

$$f^*(u_i v_{i1}) = \left\{[x*n+(i-1)]*[(n*x+n)+(i-1)]+\frac{n^2}{2}, 1 \le i \le n\right\}$$

The induced edge labels are ranging from 1 to

$[(2x*n)+n-1]*[(2x*n)+n-1]+\frac{n^2}{2}$, *if n is even*

$[(2x*n)+n-1]*[(2x*n)+n-1]+\frac{n^2+1}{2}$, *if n is odd*

Hence, $P_n \Theta 2P_m$ is an additive square mean graph.

For example, $P_4 \Theta 2P_3$ is an additive square mean graph as shown in figure 3.4.

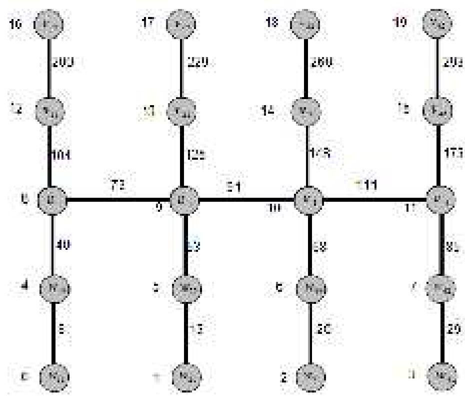

Figure 3.4

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