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Path Related Sign Graphs

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Abstract – Let G = (V,E) be a graph with p vertices and q edges. A SIGN GRAPH labeling of a graph G with vertex set V is a bijection from V to {-1, 1} such that each edge uv is assigned the label - and + with the condition that the number of vertices labeled with -1 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with **+** and **-** the number of edges labeled with 1 differ by atmost 1. The graph that admits a Sign graph lableling is called a Sign graph .In this paper, we proved that path related graphs Path P_n,$D_2(P_n)$, Z-P_n, Comb P_n^+ , Fan P_n+K_1 are Sign Graphs.

Keywords–Fan, Comp, Doublefan, Ladder, Homo-Cordial Graph, Homo-Cordial Labeling.

2000 Mathematics Subject classification 05C78.

I. Introduction

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e= {uv} of vertices in E is called edges or a line of G. In this paper, we proved that path related graphs Path P_n, $D_2(P_n)$,Z-P_n, Comb P_n^+ , Fan P_n+K_1 are Sign Graphs. are Homo-Cordial Graphs. For graph theory terminology, we follow [2].

II. Preliminaries

Let G = (V,E) be a graph with p vertices and q edges. A Homo-Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 1if f (u) =f (v) or 0 if f (u) ≠ f (v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a Homo-Cordial Labeling (HoCL) is called Homo-Cordial Graph (HoCG). In this paper, we proved that path related graphs Path P_n, $D_2(P_n)$,Z-P_n, Comb P_n^+ , Fan P_n+K are Homo-Cordial Graphs.

Definition:2.1

P_n is a path of length n-1.

Definition:2.2

Let G be a connected graph. A graph constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G to the neighbours of the corresponding vertex v in G_2, that is for every vertex u in G_1 there exists v in G_2 such that N(u)=N(v). The resulting graph is known as shadow graph and it is denoted by $D_2(G)$.

Definition:2.3

In a pair of path P_n, i^{th} vertex of a path P_n is joined with $i+1^{th}$ vertex of a path P_n It is denoted by Z-(P_n).

Definition:2.4

Let $G_1= (V_1 , E_1)$ and $G_2= (V_2 , E_2)$ be any two graphs. The join of G_1 and G_2 is the graph $G=G_1+G_2$ with vertex set $V=V_1 \cup V_2$ and edge set $E=E_1\cup E_2\cup\{uv:u\in V_1, v\in V_2\}$.

The graph P_n+K_1 is called a ***Fan***

Definition: 1.1.27

G^+ is a graph obtained from G by attaching a pendant vertex from each vertex of the graph G.

The graph P_n^+ is called comb.

III. Main Results

Theorem:3.1

Path P_n is Sign graph.

Proof:

Let $V(P_n) =\{[u_i: 1\leq i\leq n]\}$ and

$E(P_n) = \{[(u_iu_{i+1}) : 1\leq i\leq n] \}$.

Define $f:V(P_n)\rightarrow\{-1,1\}$.

The vertex labeling are ,

$$f(u_i) = \begin{cases} -1 & i \equiv 2,3 \bmod 4 \\ 1 & i \equiv 0,1 \bmod 4 \end{cases} \qquad 1 \leq \quad \leq n$$

The induced edge labeling are,

$$f^*[(u_iu_{i+1})] = \begin{cases} - & i \equiv 1 \bmod 2 \\ + & i \equiv 0 \bmod 1 \end{cases} \qquad 1 \leq \quad \leq n-1$$

Therefore, the graph P_n satisfies the conditions

$$|v_f(1) - v_f(-1)| = \begin{cases} 0 & n \text{ is odd} \\ 1 & n \text{ is even} \end{cases}$$

$$|e_f(-) - e_f(+)| = \begin{cases} 0 & n \text{ is odd} \\ 1 & n \text{ is even} \end{cases}$$

Therefore, Path P_n satisfies the conditions $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$.

Hence ,P_n is a Sign graph

For example, the Sign labeling of P_6 is shown in figure 3.2.

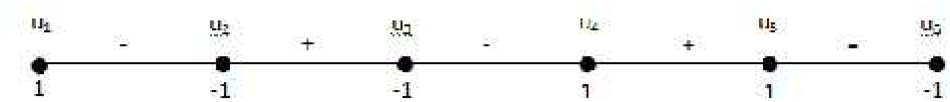

Figure 3.2

Proof:

Let $V(D_2(P_n)) = \{[u_i, v_i : 1\leq i\leq n]\}$ and

$E(D_2(P_n)) = \{[(u_iu_{i+1}) : 1\leq i\leq n-1] \cup [(u_iv_{i+1}) : 1\leq i\leq n-1] \cup [(v_iu_{i+1}) : 1\leq i\leq n-1] \cup [(v_iv_{i+1}) : 1\leq i\leq n-1]\}$.

Define $f: V(G) \rightarrow \{-1,1\}$.

The vertex labeling are ,

$$f(u_i) = \begin{cases} -1 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} -1 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are,

$f^*[(u_iu_{i+1})] = -$ $\quad 1\leq i\leq n-1$

$f^*[(v_iv_{i+1})] = +$ $\quad 1\leq i\leq n-1$

$f^*[(u_iv_{i+1})] = -$ $\quad 1\leq i\leq n-1$

$f^*[(v_iu_{i+1})] = +$ $\quad 1\leq i\leq n-1$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(0) = e_f(1)$ for all n.

Therefore, $D_2(P_n)$ satisfies the conditions $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$.

Hence, $D_2(P_n)$ is Sign graph.

For example, the Sign labeling of $D_2(P_5)$ is shown in figure 3.4

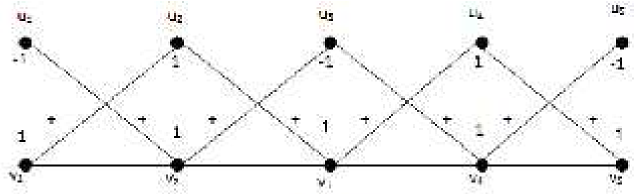

Figure 3.4

Theorem:3.5

Z - P_n is Sign graph.

Proof:

Let V(Z - P_n) ={[u_i,v_i : $1 \le i \le n$]} and

E(Z - P_n) = {[(u_iu_{i+1}): $1 \le i \le n-1$] ∪ [(v_iv_{i+1}): $1 \le i \le n-1$] ∪ [(v_iu_{i+1}): $1 \le i \le n-1$]}.

Define f : V(Z - P_n) → {-1,1}.

The vertex labeling are ,

$$f(u_i) = \begin{cases} -1 & i \equiv 1,2 \bmod 4 \\ 1 & i \equiv 0,3 \bmod 4 \end{cases} \quad 1 \le i \le n$$

$$f(v_i) = \begin{cases} -1 & i \equiv 1,2 \bmod 4 \\ 1 & i \equiv 0,3 \bmod 4 \end{cases} \quad 1 \le i \le n$$

The induced edge labeling are,

$$f^*[(u_iu_{i+1})] = \begin{cases} - & i \equiv 1 \bmod 2 \\ + & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

$$f^*[(v_iv_{i+1})] = \begin{cases} - & i \equiv 1 \bmod 2 \\ + & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

$$f^*[(v_iu_{i+1})] = \begin{cases} - & i \equiv 0 \bmod 2 \\ + & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

Here, $v_f(1) - v_f(-1) = 0$ for all n,

$$|e_f(-) - e_f(+)| = \begin{cases} 0 & \text{n is odd} \\ 1 & \text{n is even} \end{cases}$$

Therefore, Z - P_n satisfies the conditions $|v_f(0)-v_f(1)| \le 1$ and $|e_f(0)-e_f(1)| \le 1$.

Hence, Z - P_n is sign graph.

For example, the sign graph labeling of Z-P_6 is shown in figure 3.6

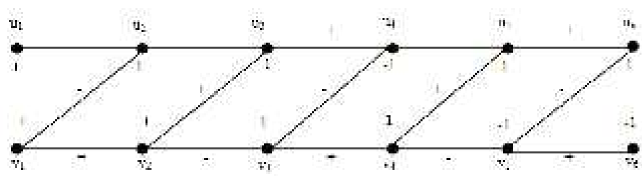

Figure 3.6

Theorem:3.7

Comb P_n^+ is Sign graph.

Proof:

Let $V(P_n^+) = \{[u_i\, v_i : 1 \le i \le n]\}$ and .

$E(P_n^+) = \{[(u_i u_{i+1}) : 1 \le i \le n-1, (u_i v_i) : 1 \le i \le n]\}$.

Define $f : V(P_n^+) \rightarrow \{-1, 1\}$.

The vertex labeling are ,

$f(u_i) = 1$

$f(v_i) = -1$

The induced edge labeling are,

$f^*[(u_i u_{i+1})] = +$

$f^*[(u_i v_i)] = -$

Therefore, P_n^+ satisfies the conditions

$|v_f(1) - v_f(-1)| = 0$ for all n

$|e_f(-) - e_f(+)| = 1$ for all n P_n^+

Hence , P_n^+ is a Sign graph

For example, the sign graph labeling of P_6^+ is shown in figure 3.8.

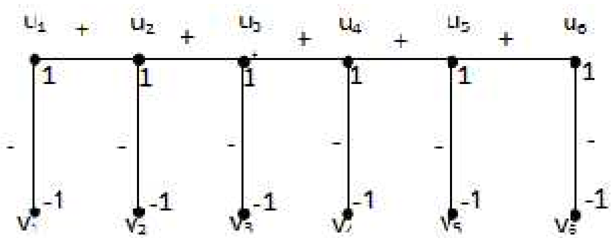

Figure 3.8

Theorem:3.9

Fan $P_n + K_1$ is Sign graph.

Proof:

Let $V(P_n + K_1) = \{[u, u_i : 1 \le i \le n]\}$ and

$E(P_n + K_1) = \{[(u u_i) : 1 \le i \le n] \cup [(u_i u_{i+1}) : 1 \le i \le n-1]\}$.

Define $f : V(P_n + K_1) \rightarrow \{-1, 1\}$.

The vertex labeling are ,

$$f(u) = 1$$

$$f(u_i) = \begin{cases} -1 & i \equiv 1,2 \bmod 4 \\ 1 & i \equiv 0,3 \bmod 4 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*[(uu_i)] = \begin{cases} - & i \equiv 1,2 \bmod 4 \\ + & i \equiv 0,3 \bmod 4 \end{cases} \quad 1 \leq i \leq n$$

$$f^*[(u_iu_{i+1})] = \begin{cases} - & i \equiv 1 \bmod 2 \\ + & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n-1$$

Here, $|v_f(1) - v_f(-1)| = \begin{cases} 0 & n \text{ is odd} \\ 1 & n \text{ is even} \end{cases}$ and

$|e_f(0) - e_f(1)| = 1$ for all n.

Therefore, Fan P_n+K_1 :(n-even) satisfies the conditions $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$.

Hence, P_n+K_1 is Sign graph.

For example, the Sign graph labeling of P_6+K_1 is shown in figure 3.10.

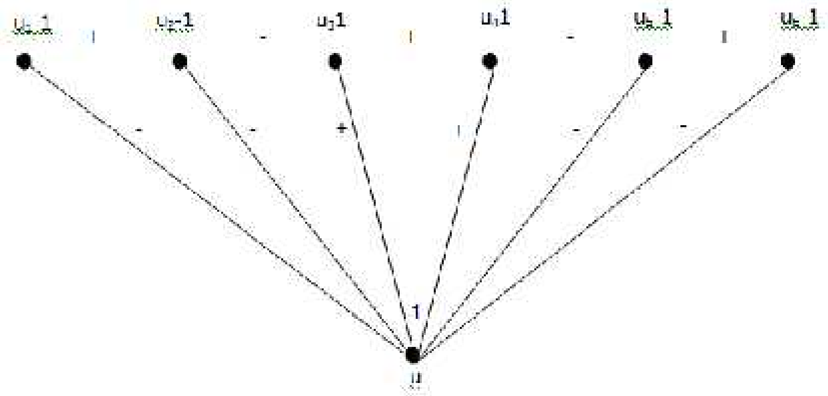

Figure 3.10

VI. References

Gallian. J.A,A *Dynamic Survey of Graph Labeling*, The Electronic Journal of Combinotorics 6(2001)#DS6.

Harary,F.(1969), *Graph Theory*, Addision – Wesley Publishing Company Inc, USA.

A.Nellai Murugan (September 2011), *Studies in Graph theory- Some Labeling Problems in Graphs and Related topics*, Ph.D Thesis.