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Results on Special Class of Hetro-Cordial Graphs

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Abstract – Let G = (V,E) be a graph with p vertices and q edges. A Hetro-Cordial labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 0 if f(u) = f(v) or 1 if f (u) ≠ f (v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. The graph that admits a Hetro-Cordial labeling is called a Hetro-Cordial Graph (HeCG). In this paper, we proved that the graphs Z-(P_n), Twig Tg_n, ($P_2 \cup mK_1$)+N_2 are Hetro-Cordial Graphs.

Keywords-Fan, Comp, Doublefan, Ladder, Hetro-Cordial Graph, Hetro-Cordial labeling.

2000 Mathematics Subject classification 05C78.

I. Introduction

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e= {uv} of vertices in E is called edges or a line of G. In this paper, we proved that the graphs Z-(P_n), Twig Tg_n, ($P_2 \cup mK_1$)+N_2 are Hetro-Cordial Graphs. For graph theory terminology, we follow [2]

II. Preliminaries

Let G = (V,E) be a graph with p vertices and q edges. A Hetro-Cordial labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 0if f (u) =f (v) or 1 if f (u) ≠ f (v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a Hetro-Cordial labeling is called a Hetro-Cordial Graph (HCG). In this paper, we proved that the graphs Z-(P_n), Twig Tg_n, ($P_2 \cup mK_1$)+N_2 are Hetro-Cordial Graphs.

Definition: 2.1

In a pair of path P_n, i^{th} vertex of a path P_n is joined with $i+1^{th}$ vertex of a path P_n. It is denoted by Z-(P_n).

Definition: 2.2

A graph obtained from a path by attaching exactly two pendant edges to each internal vertex of the path is called a twig and is denoted by Tg_n , $n \geq 1$.

Definition: 2.3

The graph $(P_2 \cup mK_1)+N_2$ is a graph with vertex set $\{z_1,z_2,x_1,x_2 \ldots x_m\} \cup \{y_1,y_2\}$ and edge set $\{[(y_1,z_1),(y_1,z_2),(y_2,z_1), (y_2,z_2), (z_1,z_2)] \cup [(y_1x_i) \cup (y_2x_i) : 1 \leq i \leq m]\}$.

III.Main Results

Theorem: 3.1

$Z - P_n$ is a Hetro-Cordial Graph.

Proof:

Let $V(Z - P_n) =\{[u_i,v_i : 1 \leq i \leq n]\}$ and

$E(Z - P_n) = \{[(u_iu_{i+1}):1 \leq i \leq n-1] \cup [(v_iv_{i+1}):1 \leq i \leq n-1] \cup [(v_iu_{i+1}) : 1 \leq i \leq n-1]\}$.

Define $f : V(Z - P_n) \rightarrow \{0,1\}$.

The vertex labeling are ,

$$f(u_i) = \begin{cases} 0 & i \equiv 0,3 \bmod 4 \\ 1 & i \equiv 1,2 \bmod 4 \end{cases} \qquad 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 1,2 \bmod 4 \\ 1 & i \equiv 0,3 \bmod 4 \end{cases} \qquad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*[(u_iu_{i+1})] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \qquad 1 \leq i \leq n-1$$

$$f^*[(v_iv_{i+1})] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \qquad 1 \leq i \leq n-1$$

$$f^*[(v_iu_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \qquad 1 \leq i \leq n-1$$

Here, $v_f(1) = v_f(0)$ for all n,

$e_f(1) = e_f(0)$ for all $n \equiv 1 \bmod 2$ and

$e_f(0) = e_f(1)+1$ for all $n \equiv 0 \bmod 2$.

Therefore, $Z - P_n$ satisfies the conditions $| v_f(0) - v_f(1) | \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$.

Hence, $Z - P_n$ is a Hetro-Cordial Graph.

For example, Hetro Cordial labeling of $Z-P_5$ and $Z-P_6$ are shown in the following fig 3.2 and fig 3.3 respectively.

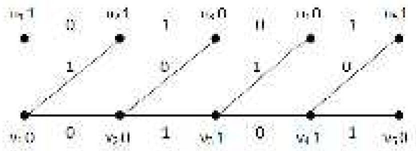

Figure 3.2: Z-P_5

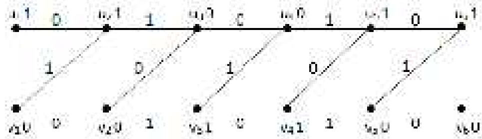

Figure 3.3: Z-P_6

Theorem: 3.4

Twig Tg_n is a Hetro-Cordial Graph.

Proof:

Let $V(Tg_n) = \{[u_i : 1 \le i \le n], [v_i, w_i : 1 \le i \le n-2]\}$ and

$E(Tg_n) = \{[(u_i u_{i+1}) : 1 \le i \le n-1] \cup [(u_{i+1} w_i) \cup (u_{i+1} v_i) : 1 \le i \le n-2]\}$

Define $f : V(Tg_n) \to \{0,1\}$.

The vertex labeling are ,

$$f(u_i) = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \le i \le n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n-2$$

$$f(w_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n-2$$

The induced edge labeling are,

$$f^*[(u_i u_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

$$f^*[(u_{i+1} v_i)] = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \le i \le n-2$$

$$f^*[(u_{i+1} w_i)] = \begin{cases} 0 & i \equiv 2,3 \bmod 4 \\ 1 & i \equiv 0,1 \bmod 4 \end{cases} \quad 1 \le i \le n-2$$

Here, Twig Tg_n satisfies the conditions $| v_f(0) - v_f(1) | \le 1$ and $| e_f 0) - e_f(1) | \le 1$.

Therefore ,Twig Tg_n is a Hetro-Cordial Graph.

For example, Hetro Cordial labeling of Tg_5 and Tg_6 are shown in the following fig 3.5 and fig 3.6 respectively.

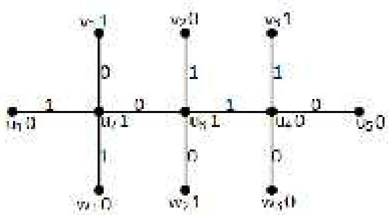

fig 3.5: Tg_5

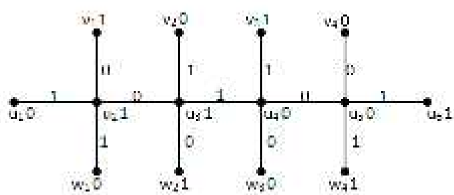

Fig 3.6: Tg_6

Theorem: 3.7

$(P_2 \cup nK_1)+N_2$ is a Hetro-Cordial Graph.

Proof:

Let $V(G) = \{[x_i : 1 \le i \le n], [y_1, y_2, z_1, z_2]\}$ and

$E(G) = \{[(y_1,z_1),(y_1,z_2),(y_2,z_1),(y_2,z_2),(z_1,z_2)] \cup [(y_1x_i) \cup (y_2x_i) : 1 \le i \le n]\}$. Define $f : V(G) \rightarrow \{0,1\}$.

The vertex labeling are ,

$f(z_1) = 0$

$f(z_2) = 1$

$f(y_1) = 0$

$f(y_2) = 1$

$$f(x_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

The induced edge labeling are,

$f^*[(y_1,z_1)] = 0$

$f^*[(y_1,z_2)] = 1$

$f^*[(y_2,z_1)] = 1$

$f^*[(y_2,z_2)] = 0$

$f^*[(z_1,z_2)] = 1$

$$f^*[(y_1x_i)] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

$$f^*[(y_2x_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \qquad 1 \le i \le n$$

Here, $v_f(0) = v_f(1)+1$ for $n \equiv 1 \bmod 2$,

$v_f(0) = v_f(1)$ for $n \equiv 0 \bmod 2$

$e_f(0) = e_f(1)+1$ for all n and

$e_f(1) = e_f(0)+1$.

Therefore, $(P_2 \cup nK_1)+N_2$ satisfies the conditions $| v_f(0) - v_f(1) | \le 1$ and $| e_f(0) - e_f(1)| \le 1$.

Hence , $(P_2 \cup nK_1)+N_2$ is a Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of $(P_2 \cup 3K_1)+N_2$ and $(P_2 \cup 4K_1)+N_2$ are shown in the following fig 3.8 and fig 3.9 respectively.

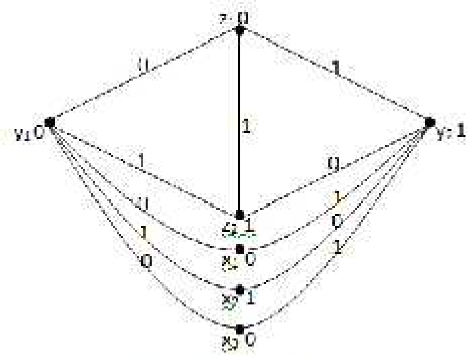

fig 3.8:$(P_5 \cup 3K_2)+N_2$

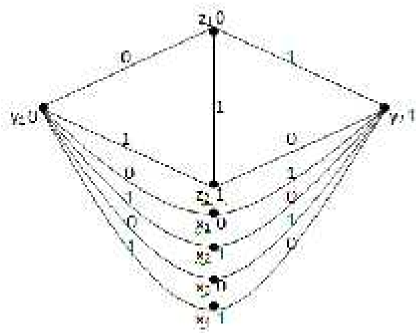

fig 3.9: $(P_5 \cup 4K_1)+N_2$

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