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Relaxed Cordial Graphs

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Abstract – Let G = (V,E) be a graph with p vertices and q edges. A Relaxed Cordial Labeling of a Graph G with vertex set V is a bijection from V to {-1 , 0, 1} such that each edge uv is assigned the label 1if |f (u) + f (v) | = 1 or 0 if | f (u) + f (v) | = 0 with the condition that the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. The graph that admits a Relaxed Cordial Labeling (RCL) is called Relaxed Cordial Graph (RCG). In this paper, we proved that $P_n \Theta C_3$, Helm W_n^+ are special Class of Relaxed Cordial Graphs.

Keywords – Helm , Relaxed Cordial Graph, Relaxed Cordial Labeling.

2000 Mathematics Subject classification 05C78.

I. Introduction

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges.Eachpair e ={uv} of vertices in E is called edges or a line of G. In this paper,we proved that $P_n \Theta C_3$, Helm W_n^+ are Relaxed Cordial Graphs. For graph theory terminology , we follow [2].

II. Preliminaries

Let G = (V,E) be a graph with p vertices and q edges. A Relaxed Cordial Labeling of a Graph G with vertex set V is a bijection from from V to {-1 , 0, 1} such that each edge uv is assigned the label 1if |f (u) + f (v) | = 1or 0 if | f (u) + f (v) | = 0 with the condition that the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a Relaxed Cordial Labeling (RCL) is called Relaxed Cordial Graph (RCG). In this paper, we proved that $P_n \Theta C_3$, Helm W_n^+ are Relaxed Cordial Graphs.

Definition: 2.1

$[P_n:C_m]$ is a graph obtained from a path P_n by joining one vertex of a cycle C_m to every vertex of a path P_n.

Definition:2.2

Helm is a graph obtained from a wheel W_n by attaching a pendant vertex from each vertex of the wheel. It is denoted by W_n^+. Here A Wheel on n (n > 4) vertices is a graph obtained from a cycle C_n by adding a new vertex and edges joining it to all the vertices of the cycle. The new edges are called the spokes of the Wheel. Also $W_n = C_n + K_1$ (n > 4).

III Main Results

Theorem : 3.1

Helm W_n^+ (even) is Relaxed Cordial Graph .

Proof:

Let the graph be a W_n^+ (n –even)

Let $V(W_n^+) = \{u, [vi,wi : 1 \leq i \leq n]\}$

$E(W_n^+) = \{[(uv_i) : 1 \leq i \leq n] \cup [(v_iv_{i+1}) : 1 \leq i \leq n1] \cup (v_1v_n) \cup [u_iu_{i+1} : 1 \leq i \leq n]\}$

Define $f : V(W_n^+) \rightarrow \{-1, 0, 1\}$

The vertex labeling are

$f(u) = 0$

$$f(v_i) = \begin{cases} 1 & i \equiv 0 \pmod 2 \\ -1 & i \equiv 1 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f(w_i) = \begin{cases} 1 & i \equiv 1 \pmod 2 \\ -1 & i \equiv 0 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are

$f^*(v_iv_{i+1}) = 0 \quad 1 \leq i \leq n-1$

$f^*(uv_i) = 1 \quad 1 \leq i \leq n$

$f^*(v_1v_n) = 0$

$$f^*(v_iw_i) = \begin{cases} 1 & i \equiv 0 \pmod 2 \\ 0 & i \equiv 1 \pmod 2 \end{cases} \quad 1 \leq i \leq n$$

Therefore, the graph $W_n$satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Hence Helm w_n^+ is Relaxed Cordial Graph

For example , the Relaxed Cordial labeling of w_6^+ is shown in the figure 3.2

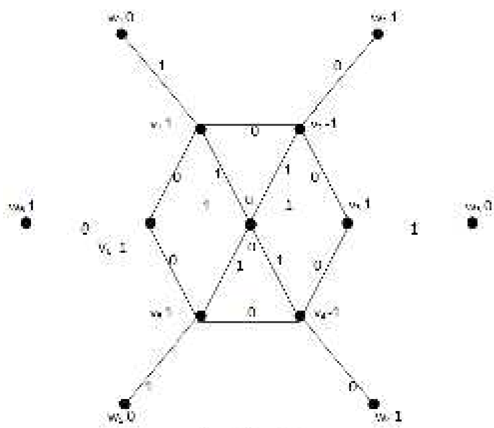


figure 3.2 w_6^+

Theorem : 3.3

Helm W_n^+ (n – odd) is Relaxed Cordial Graph.

Proof:

Let the graph be a wheel W_n^+ (n– odd)

Let $V(W_n^+) = \{ u, v_i , w_i : 1 \leq i \leq n\}$

Let $E(W_n^+) = \{[(uv_i) : 1 \leq i \leq n] \cup [(v_iv_{i+1}) \ 1 \leq i \leq n-1] \cup (v_1v_n) \cup$

$[(v_iw_i) : 1 \leq i \leq n] \}$

Define $f : V(W_n^+) \rightarrow \{ -1, 0, 1\}$

The vertex labeling are

$f(u) = 0$

$$f(v_i) = \begin{cases} 0 & i = 1, n \\ 1 & i = 0 \ (\text{mod } 2) \\ -1 & i = 1 \ (\text{mod } 2) \end{cases} \quad 1 \leq i \leq n$$

$$f(w_i) = \begin{cases} 0 & i = 1, n \\ 0 & i = 0 \ (\text{mod } 2) \\ 1 & i = 1 \ (\text{mod } 2) \end{cases} \quad 1 \leq i \leq n$$

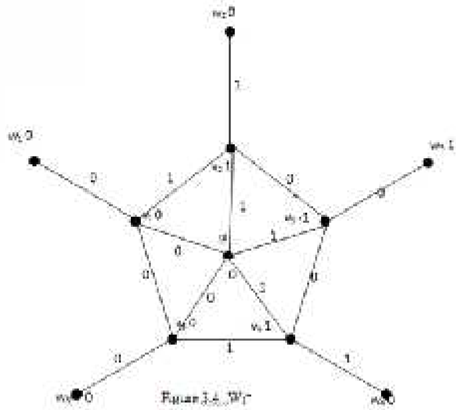
The induced edge labeling are

$$f^*(uv_i) = \begin{cases} 0 & i = 1, n \\ 1 & 2 \leq i \leq n-1 \end{cases}$$

$$f^*(v_iv_{i+1}) = \begin{cases} 1 & i = 1, n-1 \\ 0 & 2 \leq i \leq n-1 \end{cases}$$

$$f^*(v_1v_n) = 0$$

$$f^*(v_iw_i) = \begin{cases} 0 & i = 1, n \\ & i = 0 \ (\text{mod } 2) \\ 0 & i = 1 \ (\text{mod } 2) \end{cases} \quad 1 \leq i \leq n$$



Therefore , the graph W_n^+ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence ,W_n^+(n -odd) is Relaxed Cordial Graph .

For example, the Relaxed Cordial labeling of W_5^+ is shown in the figure 3.4

Theorem : 3.5

$P_n \Theta C_3$ is Relaxed Cordial Graph .

Proof:

Let the graph G be $P_n \Theta C_3$.

Let $V(G) = \{[u_i : 1 \leq i \leq n], [v_{i1}, v_{i2} : 1 \leq i \leq n]\}$

$E(G)= \{[u_iu_{i+1}) : 1 \leq i \leq n-1]U[(v_{i1}v_{i2}) : 1 \leq i \leq n [(u_iv_{ij}) : 1 \leq i \leq n , 1 \leq j \leq 2]\}$

Define $f : V(G) \rightarrow \{-1, 0, 1\}$

The vertex labeling are

$f(u_i) = 0 \quad 1 \leq i \leq n$

$f(v_{i1}) = -1 \quad 1 \leq i \leq n$

$f(v_{i2}) = 1 \quad 1 \leq i \leq n$

The induced edge labeling are

$f^*(u_iu_{i+1}) = 0 \quad 1 \leq i \leq n-1$

$f^*(u_iv_{ij} = 1 \quad 1 \leq i \leq n , 1 \leq j \leq 2$

$f^*(v_{i1}v_{i2}) = 0 \quad 1 \le i \le n$

$f^*(u_iv_{i2}) = 1 \quad 1 \le i \le n$

And $|e_f(0) - e_f(1)| = 1$ for all n

Therefore , it satisfies the condition $|e_f(0)-e_f(1)| \le 1$

Hence $\boldsymbol{P_n \Theta C_3}$ is Relaxed Cordial Graph.

For example, the Relaxed Cordial labeling of be $\boldsymbol{P_6 \Theta C_3}$ is shown in the figure 3.6

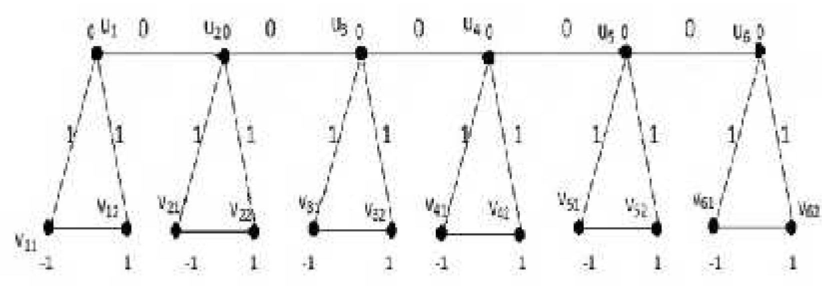


Figure 3.6: $P_6 \Theta C_3$

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