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Prime Labeling of Some Special Class Graphs

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Abstract : Prime labeling originated with Entringer and was introduced by Tout, Dabboucy and Howalla[4].A Graph G(V,E) is said to have a prime labeling if its vertices are labeled with distinct integers 1,2,3,…,|V(G)| such that for each edge xy the labels assigned to x and y are relatively prime. A graph admits a prime labeling is called a prime graph. In this paper, we prove that $P_{a,b}$ and Jelly Fish J(m,n) are prime graphs.

1. Introduction

A simple graph G= (V,E) is said to have order |V| and size |E|. A graph G is said to have a prime labeling(or called prime) if its vertices are labeled with distinct integers 1,2,3,…,|V(G)|, such that for each edge $xy \in E(G)$, the labels assigned to x and y are relatively prime[1]. The following definitions and notations are used in main results.

Definition 1.1

P_n is a path of length n. Let u and v be two fixed vertices. We connect u and v by means of "b" internally disjoint paths of length "a" each. The resulting graph is denoted by $P_{a,b}$[3].

Definition 1.2

For integers $m,n \geq 0$. We consider the graph J(m,n) with vertex set $V(\mathbf{J}(m,n))=\{u,v,x,y\}\cup\{x_1,x_2,\ldots,x_m\}\cup\{y_1,y_2,\ldots,y_n\}$ and edge set $E(\mathbf{J}(m,n))= \{(u,x),(u,v),(u,y),(v,x),(v,y)\}\cup\{(x_i,x):1\leq i\leq n\}\cup\{(y_i,y):i=1,2,\ldots,n\}$. It is known as jelly fish[5].

2. Prime Labeling of Some Special Class Graphs

Theorem 2.1 $\mathbf{P}_{a,b}$ is prime for all integers $a,b\geq 2$

Proof:

Let u and v be the fixed vertices of $P_{a,\,b}$ and w_1, $w_2 \ldots w_{b(a-1)}$ be the internal vertices of the disjoint paths of $P_{a,\,b}$.

Let G= $P_{a,b}$

$V(G) = \{u,v,w_i \,/\, 1 \leq i \leq b(a-1)\}$

$E(G)= \{uw_{i(a-1)}/1\le i\le b\}\cup\{vw_{(i-1)(a-1)+1}/1\le i\le b\}\cup\{w_iw_{i+1}/(a-1)j+1\le i\le(a-1)j+(a-2),\ j=0$ to $b-1\}$

$|V(P_{a,b})| = b(a-1)+2$

Case (i): If $a \equiv 1,3,5 \pmod 6$

Define f: $V \to \{1, 2, 3,\ldots,b(a-1)+2\}$ by

$f(u)=1$, $f(v)=2$,

$f(w_i) = i+2$, $i=1,2,3\ldots,b(a-1)$

Further,

$\gcd(f(u), f(w_{(a-1)i})) = \gcd(1,(a-1)i+2) =1$, $i=1$ to b

$\gcd(f(v), f(w_{(a-1)i+1})) = \gcd(2,(a-1)i+3) =1$, $i=0$ to $b-1$

$\gcd(f(w_i), f(w_{i+1})) = \gcd(i+2, i+3) =1$, $(a-1)j+1 \le i \le (a-1)j+(a-2)$, $j=0$ to $b-1$

Case (ii): If $a \equiv 0, 4 \pmod 6$

Define f: $V\to \{1,2,3\ldots b(a-1)+2\}$ by $f(u) = 1$, $f(v) = a-1$

$f(w_i) = i+1$, $i=1,2,3,\ldots,a-1$ except $(a-2)$

$f(w_{a-2}) = a+1$

$f(w_i) = i+2$, $i=a,a+1,\ldots b(a-1)$

Further,

$\gcd(f(v), f(w_{(a-1)i})) = \gcd(a-1,(a-1)i+2) = 1$, $i=2$ to b

$\gcd(f(u), f(w_i)) = 1$ if $i\equiv 1 \pmod{(a-1)}$ and $1 \le i \le (a-1)(b-1)+1$

$\gcd(f(w_{a-3}), f(w_{a-2})) = \gcd(a-2, a+1) = 1$

Case (iii): If $a \equiv 2 \pmod 6$ and $a\neq 2$

Define f: $V\to \{1, 2, 3,\ldots,b(a-1)+2\}$

$f(u) =1$, $f(v) = a-1$

$f(w_i) = i+1$, $i=1,2,\ldots,a-5$

$f(w_{a-1}) = a$, $f(w_{a-2}) = a+1$, $f(w_{a-3}) = a+2$,

$f(w_{a-4}) = a+3$, $f(w_a) = a-2$, $f(w_{a+1}) = a-3$,

$f(w_i) = i+2$, $i=a+2,a+3,\ldots,b(a-1)$

In this case also for every edge $uv\in E(G)$, it can be verified that $\gcd(f(u),f(v))=1$.

Case (iv): $\mathbf{P}_{2,n}$

Let $w_1, w_2,\ldots,w_n$ be the internal vertices of the path $P_{2,n}$.Then there exists a positive integer j such that $j+1$ is the greatest prime number $\le n+2$.

Define f: $V\to \{1, 2, 3,\ldots,b(a-1)+2\}$, $f(u) = 1$

***Subcase1*:** If n+2 is not a prime number

$f(w_i) = i+1,\ 1 \le i \le j-1$

$f(w_i) = i+2,\ j+1 \le i \le n$

$f(v) = j+1,\ f(w_j) = n+2$

$\gcd(f(v), f(w_i)) = \gcd(j+1, i+1) = 1 \qquad , 1 \le i \le j-1$

$\gcd(f(v), f(w_i)) = \gcd(j+1, i+2) = 1 \qquad , j+1 \le i \le n$

$\gcd(f(v), f(w_i)) = \gcd(j+1, n+2) = 1$

***Subcase2*:** If n+2 is a prime number

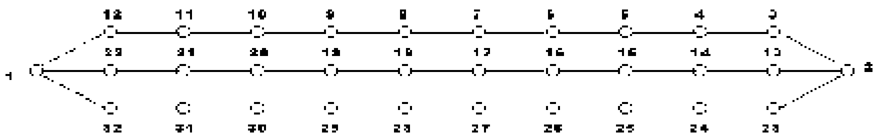
$f(w_i) = i+1, 1 \le i \le n$

$f(v) = n+2$

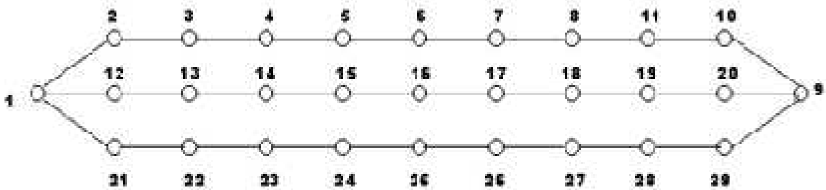
$\gcd(f(v), f(w_i)) = \gcd(n+2, i+1) = 1,\ 1 \le i \le n$

Therefore, $P_{a,b}$ admits prime labeling and hence, $P_{a,b}$ is a prime graph.

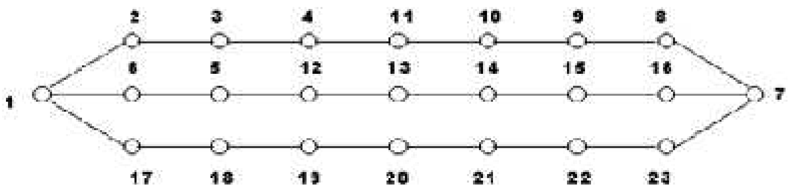
Example 2.2



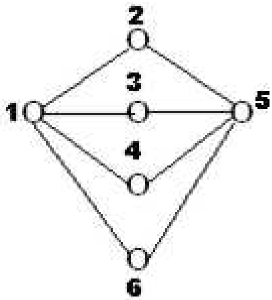
Case (i) Fig1 : $P_{11,3}$



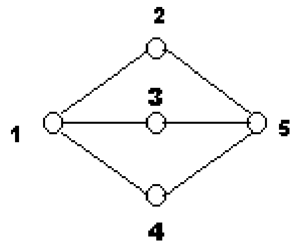
Case (ii) Fig2: $P_{10,3}$



Case (iii) Fig 3: $P_{8,3}$



Subcase(i) Fig4: $P_{2,4}$



Subcase(ii) Fig 5: $P_{2,3}$

Theorem 2.3 Jelly Fish **J(*m*,*n*)** is prime

Proof:

The graph J(m,n) has m+n+4 vertices and m+n+5 edges.

V= $\{w,x,y,z\} \cup \{u_i / 1 \le i \le m\} \cup \{v_i / 1 \le i \le n\}$

E=$\{(w,x),(x,y),(x,z),(y,z),(w,z)\} \cup \{(w,u_i)/ 1 \le i \le m\} \cup \{(y,v_i)/ 1 \le i \le n\}$

Define f: V $\rightarrow$ {1, 2…m+n+4} by

f (u_1)=2, f(u_2)=4, f(u_i)=i+3, $3 \le i \le m$

f(w)=1, f(x)=3, f(z)=5

Suppose m+n+4 is a prime number then f(y) =m+n+4=j, f(v_i)=m+3+i, $1 \le i \le n$.

Suppose m+n+4 is not prime.

Let j be the greatest prime number less than m+n+4.

Then there exist a 'k' such that m+3+k=j=f(y), $1 \le k \le n$.

f(v_i)=m+3+i, $1 \le i \le k-1$

f(v_i)=m+3+i+1,$k+1 \le i \le n-1$

f(v_k)=m+n+4.

gcd(f(x),f(y)) =gcd(3,j) =1

gcd(f(y),f(z)) =gcd(j,5) =1

gcd(f(y),f(v_i)) =gcd(j,m+3+i)=1, if m+n+4 is prime

gcd(f(y),f(v_i)) =gcd(j,m+3+i)=1, $1 \le i \le n$, $i \ne k$, if m+n+4 is not a prime

gcd(f(y),f(v_k))=gcd(j,m+n+4)=1

Hence J(m,n) is a prime graph.

Example 2.4

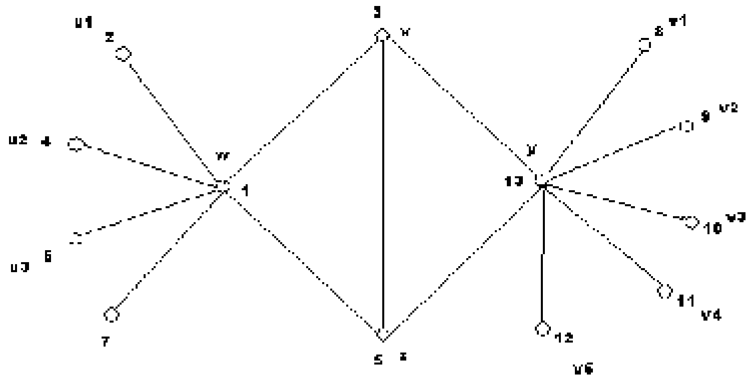


Fig 6: J (4, 5)

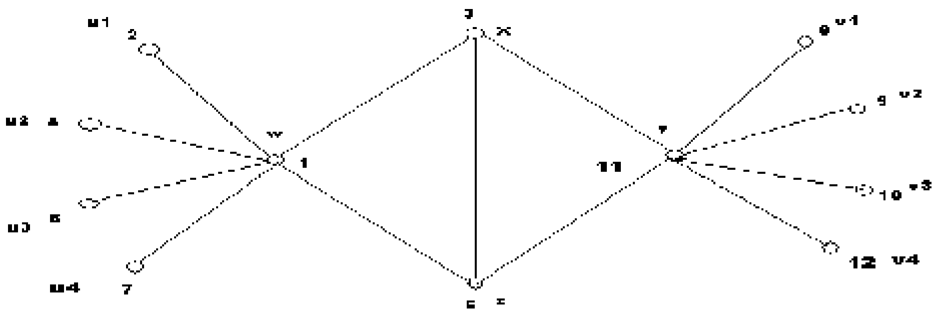


Fig 7
J(4,4)

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