OUTREACH Volume IX 2016 140 – 147
A Multi - Disciplinary Refereed Journal
ISSN (Print) : 0975 1246
ISSN (Online) : 2321 8835
www.voccoutreach.in

Homo - Cordial Snake Graphs

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Abstract : Let G = (V,E) be a graph with p vertices and q edges. A Homo-Cordial Labeling of a Graph G with vertex set V is a function from V to {0, 1} such that each edge uv is assigned the label 1if f (u) =f (v) or 0 if f (u) ≠ f (v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. The graph that admits a Homo-Cordial Labeling (HoCL) is called Homo-Cordial Graph (HoCG). In this paper, we proved that snake related graphs Quadrilateral Snake, Double Quadrilateral Snake, $(P_n \text{ X } K_2)\Theta K_1$ are Homo-Cordial Graphs.

Keywords–Snake, Homo-Cordial Graph, Homo-Cordial Labeling.

2000 Mathematics Subject classification 05C78.

I. Introduction

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e= {uv} of vertices in E is called a edge or a line of G. In this paper, we proved that snake related graphs Quadrilateral Snake Q_{n-1}, Double Quadrilateral Snake DQ_{n-1}, $(P_n \text{ X } K_2)\Theta K_1$ are Homo-Cordial Graphs. For graph theory terminology, we follow [2].

II. Preliminaries

Let G = (V,E) be a graph with p vertices and q edges. A Homo-Cordial Labeling of a Graph G with vertex set V is a function from V to {0, 1} such that each edge uv is assigned the label 1if f (u) =f (v) or 0 if f (u) ≠ f (v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a Homo-Cordial Labeling (HoCL) is called Homo-Cordial Graph (HoCG). In this paper, we proved that snake related graphs Quadrilateral Snake, Double Quadrilateral Snake,

$(P_n \text{ X } K_2)\Theta K_1$ are Homo-Cordial Graphs.

Definition: 2.1

A Quadrilateral Snake is obtained from the path $(v_1,v_2,\ldots\ldots,v_n)$ by replacing every edge by a cycle C_4. It is denoted by Q_{n-1}.

Definition: 2.2

A Double Quadrilateral Snake is obtained from the path $(v_1,v_2,\ldots\ldots,v_n)$ by replacing every edge by $2C_4$. It is denoted by DQ_{n-1}.

Definition: 2.3

The corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1(which has P_1 points) and P_1 copies of G_2 and joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2.

III. Main Results

Theorem:3.1

Quadrilateral Snake Q_n is a Homo-Cordial Graph.

Proof:

Let $V(Q_n) = \{[u_i: 1 \le i \le n+1] , [u_{i1}, u_{i2} : 1 \le i \le n]\}$ and

$E(Q_n) = \{[(u_i u_{i+1}) \cup (u_i u_{i1}) \cup (u_{i+1} u_{i2}) \cup (u_{i1} u_{i2}) : 1 \le i \le n]\}$

Define $f : V(Q_n) \to \{0,1\}$.

The vertex labeling are ,

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n+1$$

$$f(u_{i1}) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n$$

$$f(u_{i2}) = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n$$

The induced edge labeling are,

$f^*[(u_i u_{i+1})] = 0 \quad 1 \le i \le n$

$f^*[(u_i u_{i1})] = 1 \quad 1 \le i \le n$

$f^*[(u_{i+1} u_{i2})] = 1 \quad 1 \le i \le n$

$f^*[(u_{i1} u_{i2})] = 0 \quad 1 \le i \le n$

Here, $v_f(0) = v_f(1)$ for all $n \equiv 0 \bmod 2$,

$v_f(1) = v_f(0)+1$ for all $n \equiv 1 \bmod 2$ and

$e_f(0) = e_f(1)$ for all n.

Therefore, the graph Q_n satisfies the conditions $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$.

Hence, Quadrilateral Snake Q_n is a Homo-Cordial Graph.

For example, the Homo-Cordial Labeling of Q_3 and Q_4 are shown in figure 3.2 and figure 3.3

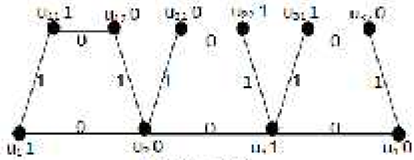

Figure 3.2: Q_3

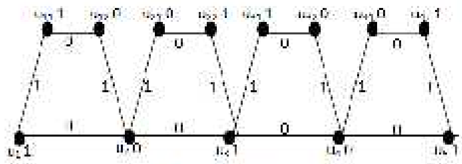

Figure 3.3: Q_4

Theorem:3.4

Double Quadrilateral Snake DQ_n is a Homo-Cordial Graph.

Proof:

Let $V(DQ_n) = \{[u_i : 1\leq i\leq n+1], [u_{i1}, u_{i2}, u_{i3}, u_{i4} : 1\leq i\leq n]\}$ and

$E(DQ_n) = \{[(u_iu_{i+1})\cup(u_iu_{i1})\cup(u_iu_{i3})\cup(u_{i+1}u_{i2})\cup(u_{i+1}u_{i4})\cup(u_{i1}u_{i2})\cup(u_{i3}u_{i4}) : 1\leq i\leq n]\}$

Define $f : V(DQ_n) \rightarrow \{0,1\}$.

The vertex labeling are ,

$$f(u_i) = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \leq i \leq n+1$$

$$f(u_{i1}) = \begin{cases} 0 & i \equiv 1,2 \bmod 4 \\ 1 & i \equiv 0,3 \bmod 4 \end{cases} \quad 1 \leq i \leq n$$

$$f(u_{i2}) = \begin{cases} 0 & i \equiv 0,3 \bmod 2 \\ 1 & i \equiv 1,2 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f(u_{i3}) = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \leq i \leq n$$

$$f(u_{i4}) = \begin{cases} 0 & i \equiv 2,3 \bmod 4 \\ 1 & i \equiv 0,1 \bmod 4 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*[(u_iu_{i+1})] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f^*[(u_iu_{i1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f^*[(u_iu_{i3})] = 1 \quad 1\leq i\leq n$$

$$f^*[(u_{i+1}u_{i2})] = 1 \quad 1\leq i\leq n$$

$$f^*[(u_{i+1}u_{i4})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$f^*[(u_{i1}u_{i2})] \quad = \quad 0 \quad 1 \le i \le n$

$f^*[(u_{i3}u_{i4})] \quad = \quad 0 \quad 1 \le i \le n$

Here, $v_f(0) = v_f(1)$ for all $n \equiv 1,3 \mod 4$,

$v_f(1) = v_f(0)+1$ for all $n \equiv 2 \mod 4$,

$v_f(0) = v_f(1)+1$ for all $n \equiv 0 \mod 4$,

$e_f(1) = e_f(0)+1$ for all $n \equiv 1,3 \mod 4$ and

$e_f(0) = e_f(1)$ for all $n \equiv 0,2 \mod 4$.

Therefore, the graph DQ_n satisfies the conditions $|v_f(0)-v_f(1)| \le 1$ and $|e_f(0)-e_f(1)| \le 1$.

Hence, Double Quadrilateral Snake DQ_n is a Homo-Cordial Graph.

For example, the Homo-Cordial Labeling of DQ_3 and DQ_4 are shown in figure 3.5 and figure 3.6

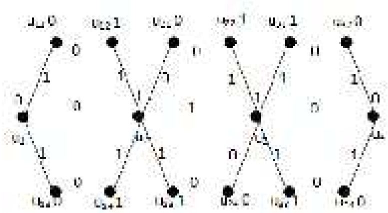

Figure 3.5: DQ_3

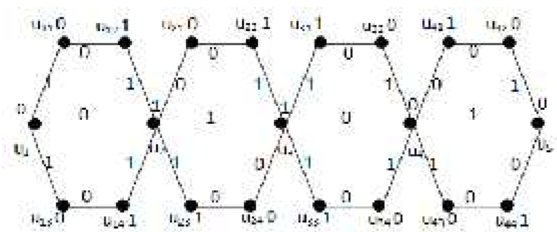

Figure 3.6: DQ_4

Theorem: 3.7

$(P_n \times K_2) \odot K_1$ (n-odd) is Homo-Cordial Graph.

Proof:

Let G be $(P_n \times K_2) \odot K_1$.

Let V (G) = $\{[u_i, v_i, x_i, y_i : 1 \le i \le n]\}$ and

E (G) = $\{[(u_iu_{i+1}) \cup (v_iv_{i+1}) : 1 \le i \le n-1] \cup [(u_ix_i) \cup (v_iy_i) \cup (u_iv_i) : 1 \le i \le n]\}$.

Define f: V (G) → {0, 1}.

Case 1:

When $n \equiv 3 \pmod 4$,

The vertex labeling are,

$f(u_i) = 0 \quad 1 \le i \le \frac{n+1}{2}$

$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad \frac{n+3}{2} \le i \le n$

$f(v_i) = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le \frac{n-1}{2}$

$f(v_i) = 1 \quad \frac{n+1}{2} \le i \le n$

$f(x_i) = 0 \quad 1 \le i \le \frac{n+1}{2}$

$f(x_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad \frac{n+3}{2} \le i \le n$

$f(y_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le \frac{n+1}{2}$

$f(y_i) = 1 \quad \frac{n+3}{2} \le i \le n$

The induced edge labeling are,

$f^*[(u_iu_{i+1})] = 1 \quad 1 \le i \le \frac{n-1}{2}$

$f^*[(u_iu_{i+1})] = 0 \quad \frac{n+1}{2} \le i \le n-1$

$f^*[(v_iv_{i+1})] = 0 \quad 1 \le i \le \frac{n-1}{2}$

$f^*[(v_iv_{i+1})] = 1 \quad \frac{n+1}{2} \le i \le n-1$

$f^*[(u_iv_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n$

$f^*[(u_ix_i)] = 1 \quad 1 \le i \le \frac{n-1}{2}$

$f^*[(u_ix_i)] = 0 \quad \frac{n+1}{2} \le i \le n$

$f^*[(v_iy_i)] = 0 \quad 1 \le i \le \frac{n+1}{2}$

$f^*[(v_iy_i)] = 1 \quad \frac{n+3}{2} \le i \le n$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(0) = e_f(1) + 1$ for all n.

Case 2:

When n≡1 (mod 4)

The vertex labeling are,

$f(u_i) = 0 \quad 1 \le i \le \frac{n+1}{2}$

$f(u_i) = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2 \end{cases} \quad \frac{n+3}{2} \le i \le n$

$f(v_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le \frac{n-1}{2}$

$f(v_i) = 1 \quad \frac{n+1}{2} \le i \le n$

$f(x_i) = 0 \quad 1 \le i \le \frac{n+1}{2}$

$f(x_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad \frac{n+3}{2} \le i \le n$

$f(y_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le \frac{n-1}{2}$

$f(y_i) = 1 \quad \frac{n+1}{2} \le i \le n$

The induced edge labeling are,

$f^*[(u_iu_{i+1})] = 1 \quad 1 \le i \le \frac{n-1}{2}$

$f^*[(u_iu_{i+1})] = 0 \quad \frac{n+1}{2} \le i \le n-1$

$f^*[(v_iv_{i+1})] = 0 \quad 1 \le i \le \frac{n-1}{2}$

$f^*[(v_iv_{i+1})] = 1 \quad \frac{n+1}{2} \le i \le n-1$

$f^*[(u_iv_i)] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n$

$f^*[(u_ix_i)] = 1 \quad 1 \le i \le \frac{n+1}{2}$

$f^*[(u_ix_i)] = 0 \quad \frac{n+3}{2} \le i \le n$

$f^*[(v_iy_i)] = 0 \quad 1 \le i \le \frac{n-1}{2}$

$f^*[(v_iy_i)] = 1 \quad \frac{n+1}{2} \le i \le n$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(1) = e_f(0) + 1$ for all n.

Therefore, the graph G satisfies the conditions $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$.

Hence, $(P_n \times K_2)\odot K_1$ (n-odd) is Homo-Cordial Graph.

For example, the Homo-Cordial Labeling of $(P_3 \times K_2)\odot K_1$ and $(P_5 \times K_1)\odot K_1$ are shown in figure 3.8 and figure 3.9 respectively.

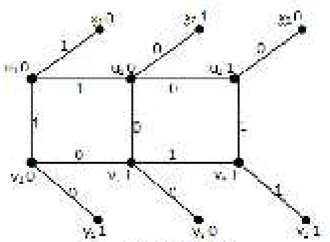

Figure 3.8: $(P_3 \times K_2)\odot K_1$

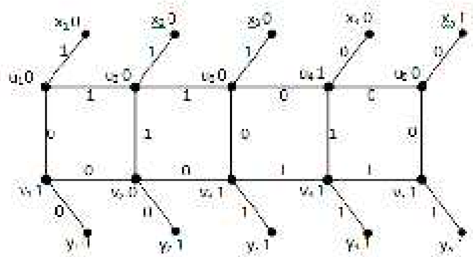

Figure 3.9: $(P_5 \times K_2)\odot K_1$

Theorem:3.10

$(P_n \times K_2)\Theta K_1$ (n-even) is Homo-Cordial Graph.

Proof:

Let G be $(P_n \times K_2)\Theta K_1$.

Let $V(G) = \{[u_i, v_i, x_i, y_i : 1 \le i \le n]\}$ and

$E(G) = \{[(u_i u_{i+1}) \cup (v_i v_{i+1}) : 1 \le i \le n-1] \cup [(u_i x_i) \cup (v_i y_i) \cup (u_i v_i) : 1 \le i \le n\}$.

Define $f: V(G) \to \{0, 1\}$.

The vertex labeling are,

$$f(u_i) = \begin{cases} 0 & i \equiv 3 \mod 4 \\ 1 & i \equiv 0,1,2 \mod 4 \end{cases} \quad 1 \le i \le n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 1,2,3 \mod 4 \\ 1 & i \equiv 0 \mod 4 \end{cases} \quad 1 \le i \le n$$

$$f(x_i) = \begin{cases} 0 & i \equiv 0 \mod 2 \\ 1 & i \equiv 1 \mod 2 \end{cases} \quad 1 \le i \le n$$

$$f(y_i) = \begin{cases} 0 & i \equiv 2,3 \mod 4 \\ 1 & i \equiv 0,1 \mod 4 \end{cases} \quad 1 \le i \le n$$

The induced edge labeling are,

$$f^*[(u_i u_{i+1})] = \begin{cases} 0 & i \equiv 2,3 \mod 4 \\ 1 & i \equiv 0,1 \mod 4 \end{cases} \quad 1 \le i \le n-1$$

$$f^*[(v_i v_{i+1})] = \begin{cases} 0 & i \equiv 0,3 \mod 4 \\ 1 & i \equiv 1,2 \mod 4 \end{cases} \quad 1 \le i \le n-1$$

$$f^*[(u_i v_i)] = \begin{cases} 0 & i \equiv 1,2 \mod 4 \\ 1 & i \equiv 0,3 \mod 4 \end{cases} \quad 1 \le i \le n$$

$$f^*[(u_i x_i)] = \begin{cases} 0 & i \equiv 0,2,3 \mod 4 \\ 1 & i \equiv 1 \mod 4 \end{cases} \quad 1 \le i \le n$$

$$f^*[(v_i y_i)] = \begin{cases} 0 & i \equiv 1 \mod 4 \\ 1 & i \equiv 0,2,3 \mod 4 \end{cases} \quad 1 \le i \le n$$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(0) = e_f(1)$ for all n.

Therefore, the graph G satisfies the conditions $|v_f(0)-v_f(1)| \le 1$ and $|e_f(0)-e_f(1)| \le 1$.

Hence, $(P_n \times K_2)\Theta K_1$ (n-even) is Homo-Cordial Graph.

For example, the Homo-Cordial Labeling of $(P_4 \times K_2)\Theta K_1$ is shown in figure 3.11

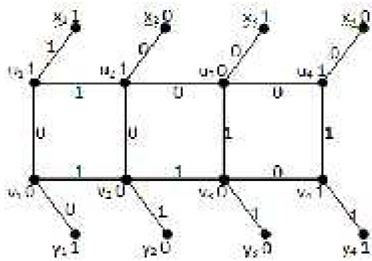

Figure 3.11: $(P_4 \times K_2)\Theta K_1$

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