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Tree Related Mean Square Cordial Graphs

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Abstract: Let G = (V,E) be a graph with p vertices and q edges. A Mean Square Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label ($\lceil (f(u))2+(f(u))2 \rceil$)2⁄ where ⌈ x ⌉ (ceilex) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a Mean Square Cordial Labeling is called Mean Square Cordial Graph. In this paper, we proved that Tree related graphs $H_n \Theta S_m$, Subdivided < $H_{n,n}$: w > are Mean Square Cordial Graphs.

Keywords – Mean Square Cordial Graph, Mean Square Cordial Labeling.

2000 Mathematics Subject classification 05C78.

I. Introduction

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {u,v} of vertices in E is called edges or a line of G. In this paper, we proved that Tree related graphs $H_n \Theta S_m$, Subdivided < $H_{n,n}$: w > are Mean Square Cordial graphs. For graph theory terminology, we follow [2].

II. Preliminaries

Let G = (V,E) be a graph with p vertices and q edges. A Mean Square Cordial Labeling of a Graph G with vertex set V is a bisection from V to {0, 1} such that each edge uv is assigned the label ($\lceil (f(u))2+(f(u))2 \rceil$)2⁄ where ⌈ x ⌉ (ceilex) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

The graph that admits a Mean Square Cordial Labeling is called Mean Square Cordial Graph. In this paper, we proved that Tree related graphs $H_n \Theta S_m$, Subdivided < $H_{n,n}$: w > are Mean Square Cordial Graphs.

Definition:2.1

The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1(which as P_1 points) and P_1 copies of $G_2$and then joining the i th point of G_1 to every point in the i th copy of G_2.

Definition:2.2

The graph $H_n \odot S_m$ is obtained from H_n by joining the root of the star to each vertex of H_n.

Definition: 2.3

H- graph H_n is a graph obtained from two copies of path P_n with vertices $(u_1,u_2,\ldots\ldots,u_n)$ and $(v_1,v_2,\ldots\ldots,v_n)$ by joining the vertices $u_{(n+1)/2}$ and $v_{(n+1)/2}$ if n is odd and $u_{n/2}$and $v_{(n/2)+1}$ if n is even.

Definition:2.4

Subdivided $< H_{n,n} : w >$ is a graph obtained from H graph while joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ if n is odd (or)$u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ if n is even through an intermediate vertex.

III. Main Results

Theorem: **3.1**

$H_n \odot S_m$ (n-even) is Mean Square Cordial Graph.

Proof :

Let G be $H_n \odot S_m$

Let $V(G) = \{ u_i , v_i : 1\leq i \leq n , u_{ij}, v_{ij} : 1\leq i \leq n, 1\leq j \leq m \}$

Let $E(G) = \{ [(u_iu_{i+1}) : 1\leq i \leq n-1] \cup [(v_iv_{i+1}) : 1\leq i \leq n-1]\cup [(u_{\frac{n}{2}}v_{\frac{n}{2}+1})]\cup$

$[(u_iu_{ij}) : 1\leq i \leq n, 1\leq j \leq m] \cup[(v_iv_{ij}) : 1\leq i \leq n, 1\leq j \leq m]\}$

Define $f : V(G) \rightarrow \{0,1\}$

The vertex labeling are,

$f(u_i) = 0 , \quad 1\leq i \leq n$

$f(u_{ij}) = 0 , \quad 1\leq i \leq n, 1\leq j \leq m$

$f(v_i) = 1 , \quad 1\leq i \leq n$

$f(v_{ij}) = 1 , \quad 1\leq i \leq n, 1\leq j \leq m$

The induced edge labeling are,

$f^*(u_iu_{i+1}) = 0 , \quad 1\leq i \leq n-1$

$f^*(v_iv_{i+1}) = 1 , \quad 1\leq i \leq n-1$

$f^*(u_{\frac{n}{2}}v_{\frac{n}{2}+1}) = 1$

$f^*(u_iu_{ij}) \quad = \quad 0 , \quad 1 \le i \le n, 1 \le j \le m$

$f^*(v_iv_{ij}) \quad = \quad 1 , \quad 1 \le i \le n, 1 \le j \le m$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(1) = e_f(0) +1$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) – v_f(0) | \le 1$

$| e_f(1) – e_f(0) | \le 1$

Hence, $H_n \Theta S_m$ (n-even) is Mean Square Cordial Graph.

For example, $H_4 \Theta S_2$ is Mean Square Cordial Graph as shown in figure3.2

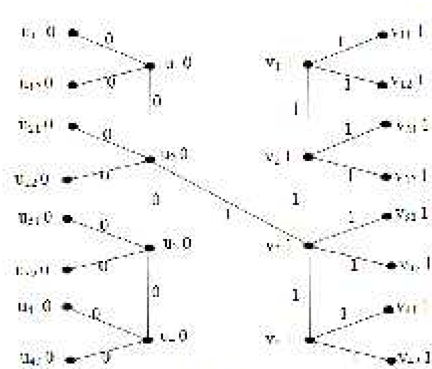

Figure 3.2

Theorem: **3.3**

$H_n \Theta S_m$ (n-odd) is Mean Square Cordial Graph.

Proof:

Let G be $H_n \Theta S_m$

Let $V(G) = \{ u_i , v_i : 1 \le i \le n , u_{ij}, v_{ij} : 1 \le i \le n, 1 \le j \le m \}$

Let $E(G) = \{ [(u_iu_{i+1}):1 \le i \le n-1] \cup [(v_iv_{i+1}) : 1 \le i \le n-1] \cup [(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}})] \cup$

$[(u_iu_{ij}) : 1 \le i \le n, 1 \le j \le m] \cup [(v_iv_{ij}) : 1 \le i \le n, 1 \le j \le m] \}$

Define $f : V(G) \rightarrow \{0,1\}$

The vertex labeling are,

$f(u_i) \quad = \quad 0 , \quad 1 \le i \le n$

$f(u_{ij}) \quad = \quad 0 , \quad 1 \le i \le n, 1 \le j \le m$

$f(v_i) \quad = \quad 1 , \quad 1 \le i \le n$

$f(v_{ij}) \quad = \quad 1 , \quad 1 \le i \le n, 1 \le j \le m$

The induced edge labeling are,

$f^*(u_iu_{i+1}) \quad = \quad 0 , \quad 1 \le i \le n-1$

$f^*(v_iv_{i+1}) \quad = \quad 1 , \quad 1 \le i \le n-1$

$f^*(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}) = 1$

$f^*(u_iu_{ij}) \quad = \quad 0 , \quad 1 \le i \le n, 1 \le j \le m$

$f^*(v_i v_{ij}) = 1, \quad 1 \leq i \leq n, 1 \leq j \leq m$

Here, $v_f(1) = v_f(0)$ for all n and

$e_f(1) = e_f(0) + 1$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) – v_f(0) | \leq 1$

$| e_f(1) – e_f(0) | \leq 1$

Hence , $H_n \odot S_m$ (n-odd) is Mean Square Cordial Graph.

For example, $H_5 \odot S_2$ is Mean Square Cordial Graph as shown in figure 3.4.

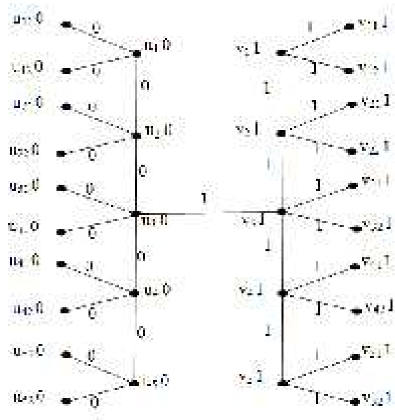

Figure 3.4

Theorem: 3.5

Subdivided $< H_{n,n} : w >$(n –even) graph is Mean Square Cordial Graph.

Proof:

Let G be Subdivided $< H_{n,n} : w >$

Let $V(G) = \{ u_i , v_i : 1 \leq i \leq n , w \}$

Let $E(G) = \{[(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(v_i v_{i+1}) 1 \leq i \leq n-1] \cup [(u_{\frac{n}{2}} w)] \cup$

$[(v_{\frac{n}{2}+1} w)]\}$

Define $f : V(G) \rightarrow \{0,1\}$

The vertex labeling are,

$f(u_i) = 0, \quad 1 \leq i \leq n$

$f(w) = 0$

$f(v_i) = 1, \quad 1 \leq i \leq n$

The induced edge labeling are,

$f^*(u_i u_{i+1}) = 0, \quad 1 \leq i \leq n-1$

$f^*(v_i v_{i+1}) = 1, \quad 1 \leq i \leq n-1$

$f^*(u_{\frac{n}{2}} w) = 0$

$f^*(v_{\frac{n}{2}+1} w) = 1$

Here, $v_f(0) = v_f(1) + 1$ for all n and

$e_f(0) = e_f(1)$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) – v_f(0) | \leq 1$

$| e_f(1) – e_f(0) | \leq 1$

Hence , Subdivided $< H_{n,n} : w >$(n –even) is Mean Square Cordial Graph.

For example, Subdivided $< H_{4,4} : w >$(n –even) is Mean Square Cordial Graph as shown in *f*igure 3.6.

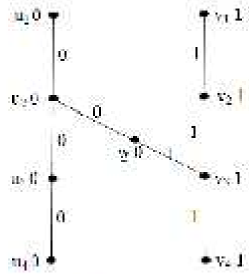

Figure 3.6

Theorem: 3.7

Subdivided $< H_{n,n} : w >$(n –odd) graph is Mean Square Cordial Graph.

Proof:

Let G be Subdivided $< H_{n,n} : w >$

Let $V(G) = \{ u_i , v_i : 1\leq i \leq n , w \}$

Let $E(G) = \{ [(u_iu_{i+1}) : 1\leq i \leq n-1] \cup [(v_iv_{i+1}) : 1\leq i \leq n-1] \cup [(u_{\frac{n+1}{2}}w)] \cup [(wv_{\frac{n+1}{2}})] \}$

Define $f : V(G) \rightarrow \{0,1\}$

The vertex labeling are,

$f(u_i) = 0 , \quad 1\leq i \leq n$

$f(w) = 0$

$f(v_i) = 1 , \quad 1\leq i \leq n$

The induced edge labeling are,

$f^*(u_iu_{i+1}) = 0 , \quad 1\leq i \leq n-1$

$f^*(v_iv_{i+1}) = 1 , \quad 1\leq i \leq n-1$

$f^*(u_{\frac{n+1}{2}}w) = 0$

$f^*(wv_{\frac{n+1}{2}}) = 1$

Here, $v_f(0) = v_f(1) + 1$ for all n and

$e_f(0) = e_f(1)$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) – v_f(0) | \leq 1$

$| e_f(1) – e_f(0) | \leq 1$

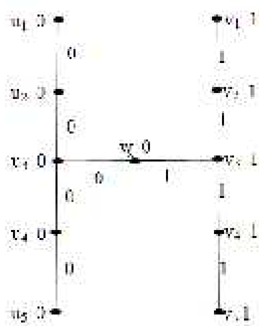

Figure 3.8

Hence , Subdivided $< H_{n,n} : w >$(n –odd) is Mean Square Cordial Graph.

For example, Subdivided $< H_{5,5} : w >$(n –odd) is Mean Square Cordial Graph as shown in figure 3.8.

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